Coefficient estimation in the dynamic equations of motion of an AUV.

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Abstract–*This article is about the development of the equations of motion in 6 DOF of the AUV (Autonomous Underwater Vehicle) SIRENA carried out by students of UPCT. As it is a pre-programmed submersible, one of the main targets is to model mathematically the submarine dynamics to obtain the simulation of its movement under any circumstance. The equations form a non-lineal system of EDPs and EDOs and are simplified using hydrodynamic coefficients. The aim of this study is to present briefly the non-lineal system of differential equations and to comment how the main coefficients have been estimated.*

Keywords *– SIRENA, AUV, hydrodynamic, maneuverability.*

I. INTRODUCTION

In the process of designing any robotic vehicle, one of the main targets is to develop the simulation of the movement. Furthermore, in expensive pre-programmed robots this aspect reaches an important role.

Nowadays, typical missions of autonomous submarines are searching mines or checking submarine wire. However, these missions need supporting boats due to low autonomy of these submersibles. One of the competitive characteristics of SIRENA is the long duration of its missions. This is due to its solar panels illustrated in the Figure 1, capable of recharging the batteries. For these missions, SIRENA would assure performing these tasks with a low logistic cost.

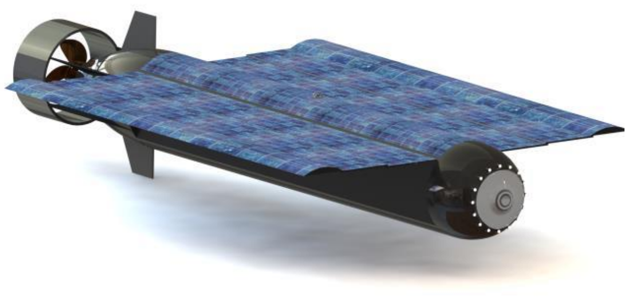

Figure 1 Panel solar

Anyway, to achieve a successful mission the movement simulation must be previously done. They dynamic model must be able to check the behaviour in case of perturbations as marine currents or artefact impacts.

Thereby, the mathematical dynamic model described in this paper provides a useful tool for understanding the movement and behaviour of SIRENA.

However; to solve these equations, coefficient estimation is an important step to establish and simplify them. Then, the mathematic algorithms would obtain the vehicle behaviour required by the project.

II. REFERENCE SYSTEM AND DEGREE OF FREEDOM

Initially, it is important to define the position, angle, moment, velocity and force components. This can be seen in the Table 1.

Table 1. Nomenclature of main parameters

DOF	Motions	Forces and moments	Linear and angular velocities	Positions and Euler angles
1	Surge	X	u	x
2	Sway	Y	v	y
3	Heave	Z	w	z
4	Roll	K	p	φ
5	Pitch	M	q	θ
6	Yaw	N	r	ψ

Other important aspect is the reference systems. Movement of AUV is described by two coordinate systems: body-fixed and earth fixed.

It's important to highlight that body-fixed system is situated in the center of a cylindrical hull.

Both reference systems and parameters previously shown in the Table 1 can be seen in the Figure 2.

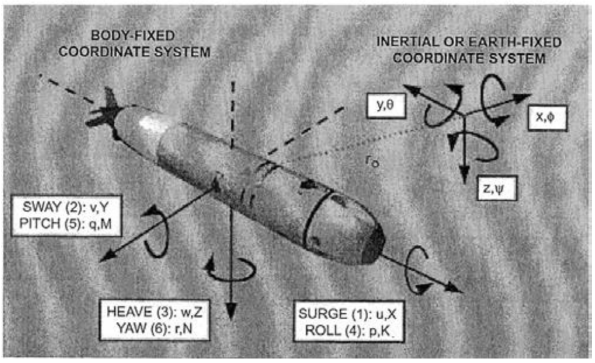

Figure 2 Reference systems

III. DYNAMIC EQUATIONS OF MOTION

Before presenting the equations, gravity centre must be defined as:

$$\overrightarrow{r_G} = \begin{bmatrix} x_g \\ y_g \\ z_g \end{bmatrix}$$

Where $\overrightarrow{r_G}$ is considered with respect to body-fixed coordinate system.

Moreover, the inertia tensor is defined as:

$$I_o = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}$$

It can be observed that it is a diagonal matrix due to the cylindrical symmetry of the submersible.

Thereby, the set of non-linear equations of the movement of the AUV can be developed as:

$$m \cdot \left[\dot{u} - vr + wq - x_g(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})\right] = \sum X_{ext}$$

$$m \cdot \left[\dot{v} - wp + ur - y_g(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(qp + \dot{r})\right] = \sum Y_{ext}$$

$$m \cdot \left[\dot{w} - uq + vp - z_g(p^2 + q^2) + x_G(rp - \dot{q}) + y_G(rq + \dot{p})\right] = \sum Z_{ext}$$

$$I_{xx}\dot{p} + \left(I_{zz} - I_{yy}\right)qr - (\dot{r} + pq)I_{xz} + (r^2 - q^2)I_{yz} + (pr - \dot{q})I_{xy} + m \cdot [y_G(\dot{w} - uq + vp) - z_G(\dot{v} - wp + ur)] = \sum K_{ext}$$

$$I_{yy}\dot{q} + (I_{xx} - I_{zz})qr - (\dot{p} + qr)I_{xy} + (p^2 - r^2)I_{xz} + (qp - \dot{r})I_{yz} + m \cdot [z_G(\dot{u} - vr + wq) - x_G(\dot{w} - uq + vp)] = \sum M_{ext}$$

$$I_{zz}\dot{r} + \left(I_{yy} - I_{xx}\right)qr - (\dot{q} + rp)I_{yz} + (q^2 - p^2)I_{xy} + (rq - \dot{p})I_{xz} + m \cdot [x_G(\dot{v} - wp + ur) - y_G(\dot{u} - vr + wq)] = \sum N_{ext}$$

Where external moments and forces can be defined as:

$$\sum F_{ext} = F_{HS} + F_{AH} + F_A + F_S + F_P$$

$$\sum M_{ext} = M_{HS} + M_{AH} + M_A + M_S + M_P$$

Being:

- Hydrostatic forces and moments: F_{HS}, M_{HS}
- Hydrodynamic damping forces and moments: F_{AH}, M_{AH}
- Added mass forces and moments: F_A, M_A
- Lift forces and moments: F_S, M_S
- Propulsion forces and moments: F_P, M_P

IV. EQUATIONS SIMPLIFICATION AND ESTIMATION COEFFICIENTS

The difficulty to integrate the previous equations is due to the external forces and moments. For example, in the hydrodynamic damping forces, it would be necessary to obtain the distribution of pressure and shear stress from momentum equations and to integrate them along the external body surface and this for each velocity value.

Hydrodynamic coefficients have an important role because of that. Including them, external forces and moments can be expressed depending from linear and angular velocity components.

With this simulator codes, which have implemented mathematics algorithms as Runge-Kutta, can integrate the equations and simulate the AUV motion.

This paper focuses on obtaining the coefficients of the main external forces: hydrodynamic forces (F_{AH}).

These external hydrodynamic forces can be expressed as:

$$X_{AH} = X_{u|u|} \cdot u|u|$$

$$Y_{AH} = Y_{v|v|} \cdot v|v| + Y_{r|r|} \cdot r|r|$$

$$Z_{AH} = Z_{w|w|} \cdot w|w| + Z_{q|q|} \cdot q|q|$$

V. DISCUSSION AND OBTAINING THE PRINCIPAL COEFFICIENTS

The coefficients estimations have been reviewed in a big variety of bibliography. The most important text about dynamic of submersible vehicle is the thesis done by Prestero [1]. Following the main texts about fluid dynamics or maneuverability, this author developed the dynamic of the movement of the commercial AUV: REMUS 100.

Thereby, mainly following this reference, the author of this paper has obtained in chapter 6 of his own grade thesis [3] all SIRENA coefficients for the external forces and moments.

Axial drag

Vehicle axial drag can be expressed by the following expression:

$$F_D = -\left(\frac{1}{2} \cdot \rho \cdot C_{D,ef} \cdot A_F\right) \cdot u|u|$$

Where ρ is the density, $C_{D,ef}$ is effective axial drag coefficient, and A_F the frontal area of SIRENA hull.

Defining a hydrodynamic coefficient as the derivate of the force respect to the velocity, the respective coefficient would be:

$$X_{u|u|} = \frac{\partial F_D}{\partial (u|u|)} = -\left(\frac{1}{2} \cdot \rho \cdot C_{D,ef} \cdot A_F\right)$$

The axial drag coefficient has been obtained in a meticulous SIRENA study done in the chapter 3 of [3].

In this case, the methodology is the one followed by Pedro Sosa [5]. The effective coefficient is estimated after obtaining the drag developed at maximum propulsion. With the drag obtained, the axial coefficient is calculated by its definition, referenced it to the frontal hull area.

Drag is defined as the sum of hull and appendix axial force.

Hull drag is the sum of pressure and viscous forces. Viscous force is obtained by ITTC friction coefficient and pressure drag is defined by form factor. This factor has been obtained from some tank experience in hulls of similar shapes and Reynolds numbers.

On the other hand, appendix drag is defined as the sum of viscous, pressure and interference drags. In this case, the author uses empirical formulas developed by Hoerner [4].

Cross flow drag

Cross flow drag is produced due to the opposition of surrounded flow when the submersible moves in lateral directions. Thus, these external forces are produced by linear and angular velocities in these directions as v, w, r and q.

Prestero, in [1], define vehicle cross flow drag as the sum of the hull cross flow drag plus the fin cross flow drag.

To simplify the obtaining method, coefficients are simplified due to cylindrical symmetry. Thereby, Y and Z forces are equals.

The cross flow drag coefficients are expressed as follows:

$$Y_{v|v|} = Z_{w|w|} = -\frac{1}{2}\rho c_{dc}\cdot\int 2R(x)\,dx - 2\cdot\left(\frac{1}{2}\rho S_{fin} c_{df}\right)$$

$$Y_{r|r|} = -Z_{q|q|} = -\frac{1}{2}\rho c_{dc}\cdot\int 2x|x|R(x)\,dx - 2x_{fin}|x_{fin}| \cdot\left(\frac{1}{2}\rho S_{fin} c_{df}\right)$$

Where c_{dc} is the drag coefficient of a cylinder, c_{df} the crossflow drag coefficient of the controls fins, S_{fin} the fin wetted surface and $R(x)$ the hull radius as a function of axial position of the submersible shape. $R(x)$ can be seen in the Figure 3.

Figure 3 Shape of SIRENA hull

Hoerner, in his text [4], provides the non-dimensional coefficients. Thus, drag coefficient of a cylinder and fins cross flow drag coefficient can be expressed as:

$$c_{dc} = 1{,}1$$

$$c_{df} = 0{,}1 + 0{,}7\cdot t_r$$

Where t_r is the relation between maximum and minimum chord of the SIRENA fins.

Coefficient values

To obtain coefficients; AUV typical values -as frontal area or surface fin- have been resolved in [3].

The results are summarized in Table 1:

Table 2 Coefficient values

Coefficient	Value	Units
X_{uu}	-1,51E+01	kg/m
Y_{vv}	-1,20E+03	kg/m
Y_{rr}	5,88E+02	kg·m/rad^2
Z_{ww}	-1,20E+03	kg/m
Z_{qq}	-5,88E+02	kg·m/ rad^2

The values are similar to those founds in literature. Coefficient in cross directions is higher, as expected, than axial coefficient due to the shape of AUV. Moreover, coefficients due to rotations are higher than translation coefficients. Thus, the results can be validated.

VI. FINAL EQUATION OF THE SIRENA MOVEMENTS AND SIMULATIONS

Following the method described by the author in reference [3], the set of non-linear simplified equations of movement in 6 DOF can be expressed as:

$$\begin{aligned}(m - X_{\dot u})\cdot\dot u + mz_G\dot q - my_G\dot r &= X_{HS} + X_{u|u|}\cdot u|u| + (X_{wq} - m)\cdot wq \\ &+ (X_{qq} + mx_G)\cdot q^2 + (X_{vr} + m)\cdot vr \\ &+ (X_{rr} + mx_g)\cdot r^2 - my_G pq - mz_G pr \\ &+ X_p\end{aligned}$$

$$\begin{aligned}(m - Y_{\dot v})\cdot\dot v - mz_G\cdot\dot p + (mx_G - Y_{\dot r})\cdot\dot r &= Y_{HS} + Y_{v|v|}\cdot v|v| + Y_{r|r|}\cdot r|r| \\ &+ (Y_{ur} - m)\cdot ur + (Y_{wp} + m)\cdot wp \\ &+ (Y_{pq} - mx_G)\cdot pq + Y_{uv}\cdot uv + my_G p^2 \\ &+ mz_G qr + Y_{uu\delta_r}\cdot uu\delta_r\end{aligned}$$

$$\begin{aligned}(m - Z_{\dot w})\cdot\dot w + my_G\cdot\dot p - (m + Z_{\dot q})\cdot\dot q &= Z_{HS} + Z_{w|w|}\cdot w|w| + Z_{q|q|}\cdot q|q| \\ &+ (Z_{uq} + m)\cdot uq + (Z_{vp} - m)\cdot vp \\ &+ (Z_{rp} - mx_G)\cdot rp + Z_{uw}\cdot uw \\ &+ mz_G(p^2 + q^2) - my_G\cdot rq + Z_{uu\delta_s} \\ &\cdot u^2\delta_s\end{aligned}$$

$$\begin{aligned}mz_G\cdot\dot v + my_G\cdot\dot w + (I_{xx} - K_{\dot p})\cdot\dot p &= K_{HS} + K_{p|p|}\cdot p|p| + K_{\dot p}\cdot\dot p + K_p\end{aligned}$$

$$mz_G \cdot \dot{u} - (mx_G + M_{\dot{w}}) \cdot \dot{w} + \left(I_{yy} - M_{\dot{q}}\right) \cdot \dot{q} = M_{HS} + M_{w|w|} \cdot w|w| + M_{q|q|} \cdot q|q| + \left(M_{uq} - mx_G\right) \cdot uq + \left(M_{vp} + mx_G\right) \cdot vp + \left[M_{rp} - \left(I_{xx} - I_{yy}\right)\right] \cdot rp + mz_G(vr - wq) + M_{uw} \cdot uw + M_{uu\delta_s} \cdot uu\delta_s$$

$$my_G \cdot \dot{u} + (mx_G - N_{\dot{v}}) \cdot \dot{v} + (I_{zz} - N_{\dot{r}}) \cdot \dot{r} = N_{HS} + N_{v|v|} \cdot v|v| + N_{r|r|} \cdot r|r| + (N_{ur} - mx_G) \cdot ur + \left(N_{wp} + mx_G\right) \cdot wp + \left[N_{pq} - \left(I_{yy} - I_{xx}\right)\right] \cdot pq - my_G(vr - wq) + N_{uv} \cdot uv + N_{uu\delta_r} \cdot uu\delta_r$$

Where δ_r and δ_s are the effective angle of SIRENA rudders in X-Z and X-Y plane.

The set of SIRENA's hydrodynamic coefficients have been obtained in the chapter 6 of reference [3]. Although this article has been focused on the coefficients of hydrodynamic damping forces, coefficient of added mass forces and moments have also a significant role, due to their importance in submersible vehicles.

From all parameters in the equations, the inputs in simulators would be effective angle, propulsion force and moment. Other parameter as numbers revolution of the helix too would be input.

The results of the simulations are shown in another publication in this Martech Workshop by others SIRENA's members. They have been obtained from a simulation tool developed in MatLab. In all graphics shown by SIRENA GUI tool, it can see how there aren't any significant mistake produced by coefficient values.

VII. REFERENCES

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